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Two Partially Filled Cavity-Resonator Techniques for the Evaluation of Scalar Permittivity and Permeability of Ferrites

The two cavity-resonator techniques described earlier for measuring pure dielectrics, one using rod samples in the cylindrical cavity system,[1] and the other using slabs in a rectangular cavity system,[2] have been extended for measuring magnetic dielectrics such as ferrites. As there are four parameters that need to be evaluated, i.e., ε_r, μ_r, $\tan_e \delta$, and $\tan_m\delta$; four independent measurements, two of the wavelengths in the partially filled portion, and two of the Q are needed. These two sets of measurements may either be obtained by using two different samples as in Srivastava's method,[3] or by using only one sample and obtaining the second set of measurements at a slightly different frequency (2 to 5 percent difference) and assuming that both μ and ε remain unaltered at this frequency. Since the cavities used in these methods are tunable, the second alternative, giving the unique advantage of using only one sample, is available.

Theory

Evaluation of ε_r and μ_r

ε_r and μ_r are obtained by measuring the two guide wavelengths in the partially filled portion by using Feenberg's method as explained in the earlier methods[1,2] for the two configurations mentioned above. The extraction of μ_r and ε_r from these measured parameters are given below.

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[1] J. K. Sinha and J. Brown, "A new cavity method for measuring permittivity," *Proc IEE* (London), vol. 107, Paper 3316E, November 1960.

[2] J. K. Sinha, "Modified technique for measuring dielectric constants using a rectangular cavity resonator," *IEEE Trans. Instrumentation and Measurements*, vol IM-16, pp. 32–48, March 1967.

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1) *Coaxial rod in a cylindrical cavity:* Maxwell's equations for this system give[1,4]

$$\beta^2 = \beta_0^2 - K_2^2 \tag{1}$$

$$= \beta_0^2\mu_r\varepsilon_r - K_1^2 \tag{2}$$

and

$$\phi(x) = \frac{1}{\mu_r}\psi(y), \tag{3}$$

where

$$\phi(x) = \frac{1}{x}\frac{J_1(x)}{J_0(x)} \tag{4}$$

and

$$\psi(y) = \frac{1}{Y}\frac{J_1(y)Y_1(my) - J_1(my)Y_1(y)}{J_0(y)Y_1(my) - J_1(my)Y_0(y)}. \tag{5}$$

In the above equations, $m=r_2/r_1$, $x=K_1r_1$, and $y=K_2r_1$, where r_1 and r_2 are the respective radii of the specimen and the cavity.

β and β_0 in the above equations refer to the propagation constants in the partially filled portion and in the free space, respectively.

Consider the equations that would be obtained for the second set of measurements, either for the same sample at a different frequency or for a different sample at the same or different frequency. By suitable manipulation one obtains

$$K_1'^2\beta_0^2 = K_1^2 + (K_2'^2\beta_0^2 - K_2^2)/\beta_0'^2 \tag{6}$$

and

$$\frac{\phi(K_1)}{\phi(K_1')} = \frac{\psi(K_2)}{\psi(K_2')}. \tag{7}$$

The primed symbols correspond to the second configuration.

[4] L. Pincherle, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," *Phys. Rev.*, vol. 66, pp. 118–130, September 1944.

K_2, $\psi(K_2)$, K_2' and $\psi(K_2')$ are now evaluated from the measured β and β'. Now, with the use of (6) and (7) one obtains an expression in K_1 or K_1' alone that can be solved numerically and, hence, μ_r and ε_r can subsequently be evaluated.

2) *Slab in the rectangular cavity:* the slab used can either be along the center or the sidewall of the cavity.[2] However, all the arguments and relations given for the previous configuration remain valid here also, except that the characteristic equations, (3)–(5), become different.

For the centrally loaded configuration one obtains

$$\phi_1(K_1t) = 2d/t\cdot\frac{1}{\mu_r}\psi_1(K_2d), \tag{8}$$

where

$$\phi_1 = \cot(K_1t/2)/(K_1t/2 \tag{9}$$

and

$$\psi_1 = \tan(K_2d)/(K_2d), \tag{10}$$

and where t is the thickness of the sample and d is the distance of the side of the sample to its nearest cavity sidewall.

For the side-loaded configuration, the corresponding characteristic equations are

$$\phi_2(K_1t) = -\frac{1}{\mu_r}\psi_2(K_2t), \tag{11}$$

where

$$\phi_2(K_1t) = \tan K_1t/K_1t \tag{12}$$

and

$$\psi_2(K_2t) = \tan K_2(a-t)/K_2t, \tag{13}$$

and where t is the sample thickness and a the broad dimension of the waveguide used in the cavity resonator.

The efforts involved in computing the

λ_2'(cms) →	PART I									λ_2' →	PART II										
λ_2(cms) ↓	3 28	3.32	3 36	3 40	3.44	3.48	3 52	3 56	3 60	λ_2 ↓	3 64	3.68	3 72	3 76	3 80	3.84	3 88	3.92	3.96	4.0	4.04
3 1724	9.72 .94	ϵ_r μ_r								35155	6.6 96		ϵ_r μ_r								
3 1731	10.02 .74									3 5163	6 8 76										
3.1739	10 3 6									3.5172	7 1 60										
3 2106		9.3 97								35534		6 3 .97									
3.2113		9.6 .76								35542		6 5 77									
3 2488		9 9 61								3 5551		6 8 61									
3.2495			8.9 98							3.5913			60 96								
3.2503			9.2 77							3 5921			6 2 73								
3.287			9 5 .62							3 5930			6.5 .61								
3 2879				8 6 .98						36291				5 7 .99							
3 2886				8.9 .73						3630				6 0 .75							
3.3252				9.2 60						3.6309				6 2 .6							
3 3259					8.3 .97					3.667					5 47 .96						
3 3267					8 5 .76					36677					5 6 76						
3 3633					8.8 .61					36686					5.9 .61						
3.3641						7 9 .99				3.7048						5 2 .96					
3 3649						8 2 .74				3.7055						5 4 .77					
3 4015						8 5 .61				37064						5 6 61					
3 4022							7 6 .96			37424							4.9 .99				
34030							7.8 75			37433							5 1 76				
34395							81 61			37442							5 3 61				
3.4403								7 3 .95		3.7801								4 6 96			
3.4410								7 5 75		3 781								4 8 74			
34774								7.8 .62		3 7819								5 1 60			
34783									6.9 .99	38177									4 4 .97		
34791									7.2 .74	3 8186									4 6 75		
									7.4 .61	38195									4 8 61		

Fig. 1. Theoretical chart relating μ_r, ε_r with λ_2, λ_2' for centrally loaded configuration (+ =0.02 inch) in the rectangular cavity. (Numbers on the top of inner squares are ε_r and on bottom μ_r.)

parameters may be obviated by plotting theoretical tables for all these configurations for the particular frequencies, such that from the experimental values of the wavelengths, the requisite parameters ε_r, μ_r may be directly read off. A typical chart for the centrally loaded rectangular configuration for the frequencies 9.1 and 9.3 kHz, for using a single sample of thickness 0.02 inch is given in Fig. 1. Many such charts for various thicknesses and frequencies may conveniently be prepared.

The limitations on the radius or the thickness of the samples discussed elsewhere[1,2] remain valid here, and it is possible to use fairly thick samples only in the centrally filled rectangular-cavity configuration,[2] such that K_2 becomes imaginary. For imaginary K_2 in this configuration, however, the procedure remains unaltered, except that K_2 is replaced by jK_2, and ψ_1 by tanh $(K_2 d)/(K_2 d)$ in the relevant equations.

1) *cylindrical cavity system*

$$\frac{1}{Q_2}-\frac{1}{Q_1}=\frac{\beta_1^2\lambda_2^2\pi^2F_1^2K_2^4\mu_r r_1[\mu_r\varepsilon_r r_1F_0\tan_e\delta+\tan_m\delta\{\mu_r\varepsilon_r F_0 r_1+2K_1J_1(K_1r_1)J_0(K_1r_1)\}]}{2L_0\lambda_1\beta_2P}; \tag{14}$$

2) *centrally loaded rectangular cavity*

$$\frac{1}{Q_2}-\frac{1}{Q_1}=\frac{\left\{\dfrac{K_2^2\mu_r t(\lambda_2)^3}{4\beta_0^2L_0\lambda_1^2}\right\}\left[K_1^2\mu_r\varepsilon_r\beta_0^2\tan_e\delta\left(1+\dfrac{K_1^2}{\mu_r^2}d^2\psi_1^2-\dfrac{2}{\mu_r t}d\psi_1\right)+\tan_m\delta\left\{G_1\left(1+\dfrac{K_1^2}{\mu_r^2}d^2\psi_1^2\right)+\dfrac{2d}{\mu_r t}\psi_1G_2\right\}\right]}{P_1}; \tag{15}$$

3) *side-loaded rectangular cavity system*

$$\frac{1}{Q_2}-\frac{1}{Q_1}=\frac{\{(\mu_rK_2^2/2\beta_0^2)(\lambda_2^3/\lambda_1^2L_0)\}[\mu_r t_r\beta_0^2\tan_e\delta(1-\phi_2+K_1^2t^2\phi_2^2)+\tan_m\delta\{G_1(1+K_1^2t^2\phi_2^2)+G_2\phi_2\}]}{P_2}. \tag{16}$$

where

$$F_0 = \left\{J_0^2(K_1r_1) + J_1^2(K_1r_1) - \frac{2}{K_1r_1} J_1(K_1r_1)J_0(K_1r_1)\right\} \tag{17}$$

$$G_1 = K_1^2 + \beta_2^2 \tag{18}$$

$$G_2 = K_1^2 - \beta_2^2 \tag{19}$$

$$P = 4K_1^2J_0^2(K_1r_1) + \pi^2r_1^2F_1^2K_2^2\left[\{F_0 - J_1^2(K_1r_1)\}\{K_2^2\mu_r - K_1^2\} + \frac{2K_1}{r_1} J_1(K_1r_1)J_0(K_1r_1)\{1 - \mu_r\}\right] \tag{20}$$

$$F_1 = J_0(K_2r_1)Y_1(K_2mr_1) - Y_0(K_2r_1)J_1(K_2mr_1) \tag{21}$$

$$P_1 = K_1^2d + \frac{K_2^2}{2} \mu_rt - \psi_1d(K_1^2 + K_2^2) + K_1^2K_2^2\psi_1^2 \frac{d(d + t)}{2\mu_r} \tag{22}$$

$$P_2 = \{K_2^2\mu_rt + (a - t)K_1^2\} - \phi_2\mu_rt(K_2^2 - K_1^2) + \phi_2^2t^2(K_1^2K_2^2)\{\mu_rt + \mu_r^2(a - t)\}. \tag{23}$$

Evaluation of $\tan_e \delta$ and $\tan_m \delta$

$\tan_e \delta$ and $\tan_m \delta$ are evaluated from measurements of Q for the two configurations. The procedure for evaluating $\tan_e \delta$ and $\tan_m \delta$ from these Q values is exactly the same as in the earlier techniques,[1,2] except that the resulting equations are a lot more complex and tedious to compute.

However, by choosing the total cavity length to be at least four half wavelengths, and if the Q of the cavity drops to at least 2/3 of its value by inserting the sample by half a wavelength, one may take certain approximations[2] which simplify the expressions very considerably and introduce an error of the order of 2 to 4 percent in the evaluation of $\tan_e \delta$ and $\tan_m \delta$. These expressions are given below. If Q_1 is the loaded Q for zero insertion, and Q_2 the loaded Q for an insertion by half a wavelength, one obtains

Conclusions

These techniques offer the possibility of very quick and accurate evaluation of permittivity and permeability of a broad range of specimen dimensions, both in the form of rods and slabs if the suitable charts are prepared. In addition, these are the only available techniques based on accurate theoretical solutions, in which one may evaluate all four parameters using only a single specimen, resulting in an enormous convenience.

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Design of TEM Equal Stub Admittance Filters

Filters formed in TEM transmission lines by short-circuited stubs that are $\lambda/4$ in length at the design center frequency and separated by the same length have useful bandpass properties in wideband (typically 10 percent to one-octave bandwidth) applications.

The usual design of a filter of this type is for a maximally flat or Chebyshev response, which requires a tapering of the characteristic admittances of the stubs [1]. The procedure described here, which may be used successfully in many applications, requires that the stub admittances all be equal. In those applications where maximum flatness or equal ripple are not required, this is a simple, inexpensive, and easily designed structure.

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The theoretical approach described here is similar to that of Mumford [1], in which we first state the form of our filter and then analyze on the basis of the exact filter model. This approach very quickly gave us the insertion loss characteristics we were seeking.

Another approach to TEM filter synthesis is to use the Richards' transformation and the Kuroda identities [2]. Various "optimum" structures in the Butterworth or Chebyshev sense have been analyzed wherein a network is synthesized to approximate a desired function. This approach has not been necessary in this case.

The analytical expressions for insertion loss are derived for any number of resonators. The assumption of dissipationless filters was made; dissipation is negligible for the relatively wideband filters considered here. Curves are available for one to eight resonators, which enables a systematic design. Bandwidth, insertion loss, and characteristic admittance may be rigorously determined in specific applications. Examples of this are given. A bandpass filter is tested and the results are shown to agree with the predictions of the theory.

Using well-known techniques, a model is analyzed for the TEM structures considered. Figure 1 shows a form suitable for the purposes of our analysis. The resonators are considered lossless. The mathematical derivation is briefly outlined here.

The filter is considered lossless, linear, passive, reciprocal, and symmetrical. The insertion loss is given by

$$L = 10 \log \left[1 - \frac{(B_N - C_N)^2}{4}\right],$$

where B_N and C_N are determined from

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}.$$

This $ABCD$ matrix is for a single (line-stub-line) section. This is then multiplied N times using techniques described elsewhere [3].

The insertion loss is given by

$$L = 10 \log \left\{1 + \frac{K^2q^2}{4(1 - q^2)} P_N^2\left[2q\left(1 + \frac{K}{1}\right)\right]\right\}, \tag{1}$$

where

1) K = characteristic admittance of stub resonator normalized to line admittance,
2) N = number of stubs,
3) $q = \cos\theta = \cos(2\pi d/\lambda)$ where d is the stub length, and
4) P_N = Chebyshev polynomial of the second kind.

This expression has been plotted for $N=1$ through 8 with K as a parameter and q as the abscissa.[1] Curves for $N=3$, 4, and 7 are shown in Figs. 2, 3, and 4, respectively.

As far as these graphs are concerned, we may immediately state the following. We have $q = \cos\theta$, so q varies as $-1 \le q \le 1$ as θ or λ varies. At $q=0$, the insertion loss is zero for all N and K. At $q = \pm 1$, the insertion loss is infinity for all N and K. In addition, P_N^2 is an even function of q, so that it is only necessary to plot the region $0 \le q \le 1$ due to symmetry. All values of λ of interest are mapped in this region.

It is seen from (1) that, for $N=1$ and $N=2$, the quarter-wave shorted-stub filter is identical to the maximally flat filter of Mumford [1].

The next section gives design examples and insertion loss curves. With the use of (1), we may easily derive approximate equations for specific regions of the insertion loss characteristic. These are useful when a curve is not available. We do not reproduce these approximations, since there are so many depending on the region of interest.

As a design example let it be required to have a bandpass TEM filter with a minimum 3-dB bandwidth of 630 MHz or 70 percent, a center frequency of 900 MHz, and a minimum rejection of 20 dB at 900±500 MHz. Each of our stubs must be $\lambda/4$ in length or 8.33 cm at 900 MHz. The 70-percent bandwidth corresponds to a 3-dB frequency of $q=0.525$ and a 20-dB frequency of $q=0.766$. Examination of the curves show that a filter of $N=4$ and $K=1.4$ (35.7-ohm stubs) will do the job. The ripple will be 0.3 dB at one point and this will be adequate for many applications.

[1] Complete graphs are available from ADI Auxiliary Publications Program.